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Problem solving 2

Graphs and Groups, Spectra and Symmetries
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1. Let Γ be a graph and C be a cycle with possible repeated vertices in Γ. Explain that is meant for C to be triangulable.

2. Suppose that Γ satisfies the following conditions for all $2 \leq i \leq diam(\Gamma)$:

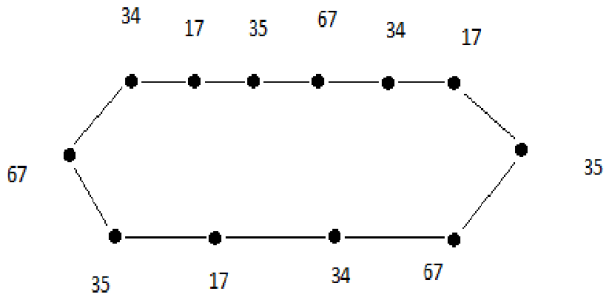
(a) whenever $u \in \Gamma_i(v)$ the subgraph included by $\Gamma_{i-1}(u) \cap \Gamma(v)$ is connected;

(b) whenever u and w are adjacent vertices in $\Gamma_i(v)$ the set $\Gamma_{i-1}(u) \cap \Gamma_{i-1}(w) \cap \Gamma(v)$ is non-empty.

Show that every cycle in Γ is triangulable.

3. Show that every cycle in the complement of the Johnson graph $J(n, 2)$ is triangulable whenever $n \geq 8$.

4. Show that the following cycle is triangulable in the complement of $J(7, 2)$.



5. A surjective map $\phi : V(\Gamma) \to V(\Psi)$ is a k-cover of graphs if $|\phi^{-1}(v)| = k$ for every $v \in V(\Psi)$ and if image of and edge of Γ is an edge of Ψ. Construct a 3-cover of the complement of $J(7, 2)$ which is locally the Petersen graph.